Thursday September 14th, 2007

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Show all necessary work in each problem to receive credit. Each problem is worth 10 points.

1. Suppose that a "word" is any string of seven letters of the alphabet, with repeated letters allowed.

a) How many words have no vowels?

Ans: $(26-5)^7 = 21^7$.

b) How many words begin with A or B or end with A or B?

Ans: $2 \cdot 26^6 + 2 \cdot 26^6 - 4 \cdot 26^5 = 4(26^6 - 26^5)$.

2. Suppose $|A| = 4$ and $|B| = 10$.

a) Find the number of functions $f : A \to B$.

Ans: 10^4.

b) Find the number of 1-1 functions $f : A \to B$.

Ans: $P(10,4) = 10 \cdot 9 \cdot 8 \cdot 7$.

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3. In the questions below suppose you have a class with 30 students: 10 freshmen, 12 sophomores, and 8 juniors.

a) In how many ways can you put all students in a line so that the freshmen are first, the sophomores are in the middle, and the juniors are at the end?

Ans: $10! \cdot 12! \cdot 8!$.

b) In how many ways can you get a committee of 4 freshmen and 3 sophomores?

Ans: C(10,4) C(12,3) or $\binom{10}{4}\binom{12}{3}$ = 46,200.

4. Use a tree diagram to find the number of binary strings of length 4 that do not contain the string 010

12 such strings

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5. A professor teaching a Discrete Math course gives a multiple choice quiz that has 4 questions, each with two possible responses: a, b. What is the minimum number of students that must be in the professor's class in order to guarantee that at least three answer sheets must be identical? (Assume that no answers are left blank.)

   Ans: There are 2^4 possible answer sheets. Therefore $(r-1)k + 1 = 2 \cdot 2^4 + 1 = 33$ is the minimum number that will guarantee three identical answer sheets.

6. You pick cards one at a time without replacement from an ordinary deck of 52 playing cards. What is the minimum number of cards you must pick in order to guarantee that you get

   (a) a pair (for example, two kings or two 5s).

   Ans: $13 + 1 = 14$.

   (b) at least two kings

   Ans: $4 \cdot 12 + 2 = 50$.

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7. A computer is programmed to print subsets of $\{1,2,3,4,5\}$ at random. If the computer prints 40 subsets, prove that some subset must have been printed at least twice.

   Ans: There are $2^5 = 32$ subsets. If 33 or more subsets are printed, at least one will have been printed twice.

8. How many permutations of the seven letters A,B,C,D,E,F,G have the two vowels before all five consonants?

   Ans: $2 \cdot 5! = 240$.

9. How many permutations of the seven letters A,B,C,D,E,F,G do not have the vowels next to each other?

   Ans: $7! - 2 \cdot 6! = 3600$.

10 How many 9 lowercase letter strings contain exactly 2 vowels?

   Ans: $C(9,2) \cdot 5^2 \cdot 21^7 = 36 \cdot 25 \cdot 21^7 = 900 \cdot 21^7$